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Corneal aberrations in keratoconic eyes: influence of pupil size and centering

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Abstract. Ocular aberrations vary among subjects and under different conditions and are commonly analyzed expanding the wavefront aberration function in Zernike polynomials. In previous articles, explicit analytical formulas to transform Zernike coefficients of up to 7th order corresponding to an original pupil into those related to a contracted displaced new pupil are obtained. In the present paper these formulas are applied to 20 keratoconic corneas of varying severity. Employing the SN CT1000 topographer, aberrations of the anterior corneal surface for a pupil of semi-diameter 3 mm centered on the keratometric axis are evaluated, the relation between the higher-order root mean square wavefront error and the index KISA% characterizing keratoconus is studied and the size and centering of the ocular photopic natural pupil are determined. Using these data and the transformation formulas, new coefficients associated to the photopic pupil size are computed and their variation when coordinates origin is shifted from the keratometric axis to the ocular pupil centre is analyzed.

1. Introduction

Visual quality depends on the performance of the optical and neural ocular systems, the first can be limited by dynamic aberrations [1,2] and the latter can remove some of their effects (the brain switching from one adaptation state into another) [3-10]. The wavefront aberration for a certain object point is traditionally expanded in Zernike polynomials, coefficients depending on each eye's structure and on pupil size and location [4]. Aberrations of the whole eye can be measured with aberrometers (employing Hartmann-Shack sensor [5,6], laser ray tracing, etc.) while those of the anterior corneal surface can be computed performing virtual ray tracing through this surface following topographic measurements [3,4,7,9]. The pupils can be considered as having the same arbitrary sizes in both cases while their centers are usually on the line of sight in the first case and on the keratometric axis in the latter. To compare ocular and corneal coefficients, a coefficients transformation, shifting pupil origin, is needed [3,7]. A coefficients transformation is also often required [3] to analyze aberrations in everyday vision since the natural ocular pupil size depends on illumination, accommodation, defocus,

psychophysical state, etc. and pupil centre moves slightly with changes in pupil size [3]. Various authors have addressed this problem proposing either analytical or numerical methods to transform coefficients [11,12], some only considering pupil scaling [13]. We have developed 2 methods to find a new set of coefficients associated to a circular pupil in terms of an original set evaluated for a dilated shifted circular pupil, one analytical (valid for a 7th order aberration expansion) and the other graphical (valid for any order of the expansion), and we have also derived selection rules [14-17]. On the other hand, aberrations of up to 2nd order are routinely compensated with conventional ophthalmic or contact lenses while the customized correction of higher-order aberrations may yield considerable visual benefit [5] in old or abnormal eyes (post-Lasik [6], suffering keratoconus [5,7,10], implanted with intraocular lenses [8], with corneal transplant [10], etc.) where the aberration balance between cornea and crystalline lens attained in young normal subjects is degraded [5,9]. A typical pathology where corneal higher-order aberrations can be very large is keratoconus [4,5,7,10], first described in 1854 [18]. This disease is a cone-like distortion of the cornea characterized by local thinning of corneal stroma leading to increase in local curvature or ectasia, it can affect both corneal surfaces (the anterior one being the most important ocular refracting element [19]) and it is usually bilateral. Though keratoconus normally starts during the second decade of life, it can also appear as a result of Lasik surgery [18] and keratoconic eyes must be excluded from refractive surgery procedures [19,20]. Keratoconus suspects may have symptoms similar to those of normal ametropes but, as the disease progresses, it can cause high myopia and irregular astigmatism which cannot be corrected with spectacles [18], distortion of vision, multiple images, visual degradation in low contrast tasks [4,19], sensitivity to light, etc. Eyes with mild to moderate keratoconus can be corrected with contact lenses either conventional (which are often rigid gas permeable and, because of the similar refraction indices of the lens, tear film and cornea, can also reduce some higher-order aberrations [19]) or customized (which can be prism ballasted soft toric and can yield better correction of spherical aberration and wearing comfort [4,10]). Eyes suffering more severe keratoconus can be treated with surgical procedures, such as keratoplasty and intra corneal rings segments, with cross-linking, etc. [18].

In the current article we apply our formulas [14-17] to transform aberration coefficients corresponding to both corneas of 10 subjects suffering mild to severe keratoconus. In Section 2 we briefly describe our methodology. In Section 3, using the SN CT1000 topographer [15], we determine the coefficients pattern for a pupil of semi-diameter 3 mm; we study the relation between higher-order root mean square wavefront error (RMS_{HO}) and the index KISA% [20] and we find the natural photopic pupil size and centering. In Section 4, using our transformation formulas and for the photopic pupil size that corresponds to each eye, we analyze the variation of RMS_{HO} when coordinates origin is shifted from the keratometric axis to the natural pupil centre.

2. Analytical method to find new aberration coefficients, subjects and methodology

The wavefront aberration for a given field point, $W(\rho, \theta)$, is [1] the optical pathlength along the ray from the reference sphere to the wavefront at the exit pupil ((ρ, θ) being normalized polar coordinates at this pupil reference sphere). If C_n^m indicate coefficients; n the order ($n_{max} \geq n \geq 0$) and m the frequency ($m = -n + 2k$ with k integer and $0 \leq k \leq n$), the expansion of $W(\rho, \theta)$ in Zernike polynomials $Z_n^m(\rho, \theta)$ is [2]

$$W(\rho, \theta) = \sum_{n=0}^{n_{max}} \sum_{m=-n}^n C_n^m Z_n^m(\rho, \theta) \quad (1)$$

and we consider $n_{max}=7$ which is sufficient to accurately describe aberrations of most eyes [3,7]. Following OSA's ordering and normalization [2] (adopted by ANSI [3]), Zernike modes are represented in a pyramid (figure 1 (a)) and, to simplify the notation [2], are henceforth indicated with the single index $j = [n(n+2)+m]/2$. The root mean square wavefront error is $RMS = (\sum C_j^2)^{1/2}$ and higher-order ocular aberrations ($j > 5$) can be regarded as negligible [14] if $RMS_{HO} < 0.10 \mu m$ (for example, the mean spherical aberration coefficient for a normal population and a pupil semi-diameter of 3mm is

$C_{12}=(0.10\pm 0.10)\ \mu\text{m}$ [3]). Though RMS_{HO} is probably not the ideal predictor of visual performance [4], for simplicity and as is usual practice [1,7], we adopt it as global optical quality metric throughout.

Considering that the values of the original coefficients C_j ($j < 36$) are known for a pupil of radius a centered at some point (the associated frame has coordinates (X, Y)), we find [14-17] new coefficients C'_j for a pupil of radius b centered at a point of coordinates (d, g) (the associated frame has coordinates (X', Y')). We have (figure 1 (b))

$$X=X'+d \quad Y=Y'+g \quad f=(d^2+g^2)^{1/2} \quad (2)$$

We consider that the relative contraction, $B=b/a$, and the relative decentering, $D=d/a$ and $G=g/a$, verify $B \leq 1$ and $|F| < 1-B$ (with $F=f/a$). Associated to (X, Y) , we define normalized cartesian coordinates $(x, y)=(X/a, Y/a)$ and corresponding polar ones (ρ, θ) with $\rho=(x^2+y^2)^{1/2}$, $\cos(\theta)=x/\rho$ and $\sin(\theta)=y/\rho$ (similarly but with primed variables for (X', Y')). The analytical expressions for the elements T_{jj} of the scaling-transverse traslation matrix, \mathbf{T} , are given in our previous paper [17] and we have

$$C'_j = \sum_{j=0}^{35} T_{jj} C_j \quad j' = 0, \dots, 35 \quad (3)$$

In what follows we determine aberrations of the anterior corneal surface in both eyes of 10 keratoconic subjects: D.R., M.A., M.O., P.A., O.A., N.A., G.A., W.A., M.J. and M.G. (KISA% ranging from 27 to 53138 and maximum corneal power [4] from 46 D to 59 D). The original coefficients C_j for a fictitious pupil of radius a centered at the keratometric axis are computed by virtual ray tracing using the SN CT1000 topographer developed by G.Martin, and described elsewhere [15] (the image plane containing the curvature centre of the reference sphere defined as the one that best fits the wavefront). The new coefficients C'_j are evaluated employing our program SLG07 which is such that the entrance data are a, b, d, g and C_j and the exit data are B, D, G, F and C'_j [17].

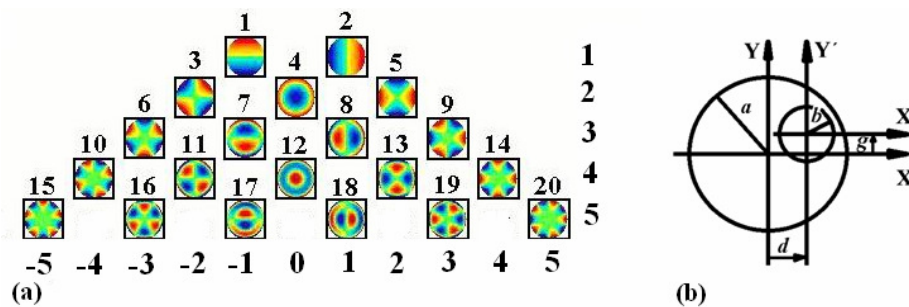


Figure 1 (a) Zernike pyramid for $j \leq 20$. (b) Pupil scaling and decentering, a : original pupil radius, b : new pupil radius, (d, g) : pupil decentering ($f=(d^2+g^2)^{1/2}$).

3. Results obtained using the SN CT1000 topographer

Using the SN CT1000 topographer to determine corneal aberrations centered at the keratometric axis for $a=3$ mm we find that, as is known [5,7,10,19], the dominant coefficients are 2nd order astigmatism and 3rd order coma, mainly $j=7$ (the largest being $C_7^{(M.J.)}=-4.85\ \mu\text{m}$ for M.J.'s right eye). The most important 4th order coefficient is sometimes astigmatism but it is more frequently spherical aberration [5,19] (the largest is $C_{12}^{(N.A.)}=-1.76\ \mu\text{m}$ for N.A.'s left eye), this aberration having a high impact on visual quality [10]. The 5th and higher-order coefficients ($j > 14$) are relatively small (the largest is $C_{17}^{(M.G.)}=0.63\ \mu\text{m}$ for M.G.'s left eye). For comparison, besides finding coefficients C_j for $a=3$ mm, with SN CT1000 we find coefficients C'_j for a pupil of semi-diameter $b=1.5$ mm. If sub-scripts indicate the values of a and b , we have $0.58\ \mu\text{m} < \text{RMS}_{\text{HO}}|_{a=3\text{mm}} < 5.77\ \mu\text{m}$ and $0.11\ \mu\text{m} < \text{RMS}_{\text{HO}}|_{b=1.5\text{mm}} < 0.99\ \mu\text{m}$ (which are within the range found by other authors [7,19]) so, for a given eye and as expected [3,4,7,19], RMS_{HO} considerably decreases if pupil semi-diameter is reduced to half its original value.

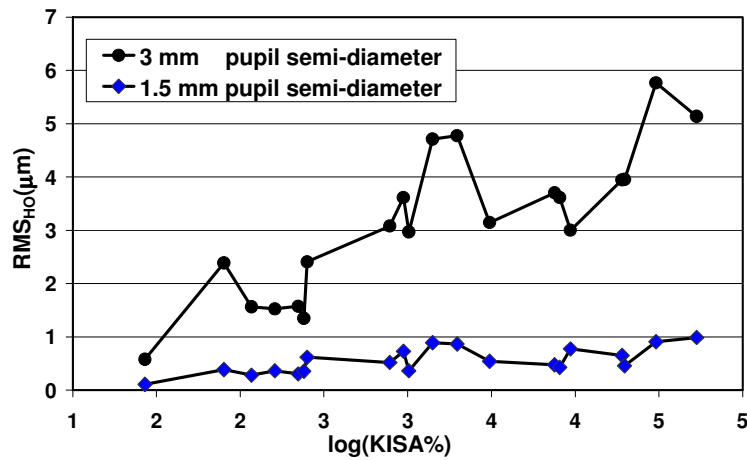


Figure 2 RMS_{HO} vs log(KISA%) for the 20 eyes and for two pupil semi-diameters: 3 mm and 1.5 mm

On the other hand, to quantify keratoconus evolution, the index KISA% (computed considering a corneal area of diameter around 7 mm) is sometimes used [18,20]. We consider that $50 < KISA\% < 100$ for suspects, $KISA\% > 100$ for clinically detectable and $KISA\% > 1000$ for advanced keratoconus. Studying the relation between RMS_{HO} and KISA% (figure 2), we obtain that, as log(KISA%) increases, RMS_{HO|a=3mm} tends to increase whereas RMS_{HO|b=1.5mm} scarcely increases (the mean is 0.55 μm and the standard deviation is 0.27 μm). Thus, though KISA% could be useful to quantify keratoconus, its relation with RMS_{HO}, under natural vision conditions, is strong for large pupils (usually encountered under non photopic conditions) but not for daylight pupils (typically of radius 1.5 to 2 mm [4]) and this is related to the fact that RMS_{HO} depends not only on keratoconus stage but also on pupil parameters. The value of RMS_{HO} for a 3 mm pupil semi-diameter has been employed by Gobbe et al [19] to establish a criterion to screen early keratoconus. Considering 28 suspected and 45 diagnosed keratoconic corneas and also 870 normal eyes, for pupil semi-diameters of 1.5; 2.25 and 3 mm, these authors find that the differences between suspected keratoconus and normal eyes are significant only for large pupils stating that the cornea shows characteristics typical of keratoconus if $RMS_{HO|a=3mm} > 0.63 \mu m$. Taking this into account and to facilitate our analysis, we organize subjects in the 3 following groups.

- **Group I** (D.R., M.A., M.O., P.A.) with $0.58 \mu m < RMS_{HO|a=3mm} < 2.97 \mu m$
- **Group II** (O.A., N.A., G.A.) with $3.00 \mu m < RMS_{HO|a=3mm} < 3.94 \mu m$
- **Group III** (W.A., M.J., M.G.) with $3.62 \mu m < RMS_{HO|a=3mm} < 5.77 \mu m$

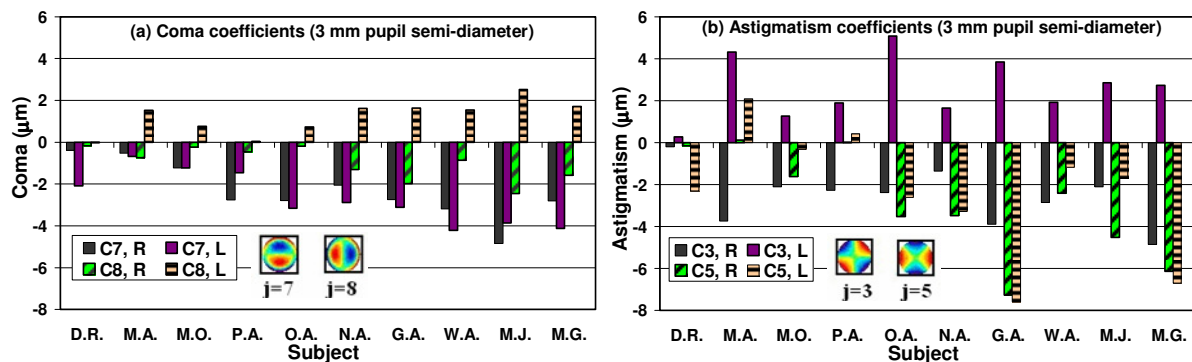


Figure 3 Dominant aberrations if $a=3$ mm for the right (R) and left (L) eyes of each subject. (a) 3rd order coma coefficients (C₇ and C₈). (b) 2nd order astigmatism coefficients (C₃ and C₅).

In figure 3 we show the dominant aberrations for the right (R) and left (L) eyes of subjects ordered according to increasing $RMS_{HO|a=3mm}$. Coma $j=7$ is, as expected [7,10], negative in both eyes while $j=8$ is positive in L and negative in R. Similarly, the sign of $j=5$ (with-the-rule astigmatism [5]) coincides in both eyes of each subject while $j=3$ is positive in L and negative in R.

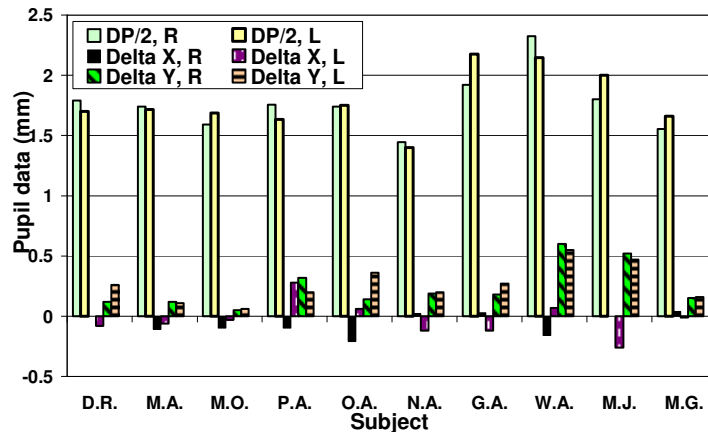


Figure 4 Natural photopic pupil semi-diameter (DP/2) and shift ($\Delta X, \Delta Y$) for the right (R) and left (L) eyes of each subject.

Since the knowledge of the natural pupil data is crucial to represent the eye's working conditions [19], when recording the corneal topography, we measure pupil diameter (DP) and the horizontal (ΔX) and vertical (ΔY) distances from the keratometric axis to pupil centre, conditions being photopic ((260 ± 20) lx in the analyzed eye). In figure 4 we plot DP/2, ΔX and ΔY for both eyes of each subject and get $1.4 \text{ mm} < DP/2 < 2.35 \text{ mm}$, the mean pupil semi-diameter being 1.8 mm. Pupil centre is above the keratometric axis with $0 < \Delta Y < 0.6 \text{ mm}$ and the shift $\delta = (\Delta X^2 + \Delta Y^2)^{1/2}$ is such that the mean is 0.3 mm and the standard deviation is 0.2 mm, this being within the range found by other authors [7].

4. Results obtained using program SLG07

Using the original coefficients (C_j) supplied by SN CT1000 for $a=3 \text{ mm}$ and program SLG07, we compute new coefficients (C'_j) for $b=DP/2$ in the following 2 situations:

- i) $d=g=0$ (coordinates origin at the keratometric axis)
- ii) $d=\Delta X$ and $g=\Delta Y$ (coordinates origin at the natural ocular pupil centre)

Situation (i) is fictitious and, for brevity, the associated coefficients are termed $C'_j|_0$ (suffix 0 indicating that $D=G=F=0$) while situation (ii) corresponds to eyes under real world photopic conditions and the related coefficients are termed $C'_j|_F$ (suffix F indicating that $F \neq 0$). In both situations, the higher-order root mean square wavefront error is considerably smaller than for the original pupil and, for example, in situation (ii) we get $RMS_{HO|F} < 0.74 \mu\text{m}$ for eyes in Group I; $RMS_{HO|F} < 1.65 \mu\text{m}$ for those in Group II and $RMS_{HO|F} < 1.94 \mu\text{m}$ for those in Group III.

As all new coefficients, the dominant higher-order one, C'_7 , depends on corneal profile and on pupil parameters (figure 5). If $O_{n \geq 5}$ indicates terms proportional to coefficients of order $n \geq 5$, we have [17]

$$C'_7|_0 = C_7 B^3 + O_{n \geq 5}|_0 \tag{4}$$

$$C'_7|_F = C'_7|_0 + C_{11} 2 (5)^{1/2} B^3 D + C_{12} 2 (10)^{1/2} B^3 G - C_{13} 2 (5)^{1/2} B^3 G + O_{n \geq 5}|_F - O_{n \geq 5}|_0$$

original coefficients C_7, C_{11} , etc. and parameters B, D and G having particular values for each eye.

Cases of advanced keratoconus can have less coma than those which are not so severe and, for example, we get the following results.

- For L of G.A. (Group II): $KISA\%^{(G.A.)} = 7540$, $DP^{(G.A.)}/2 = 1.90$ mm, $C'_{7|F}^{(G.A.)} = -1.41$ μm
- For L of M.G. (Group III): $KISA\%^{(M.G.)} = 53138$, $DP^{(M.G.)}/2 = 1.65$ mm, $C'_{7|F}^{(M.G.)} = -1.29$ μm

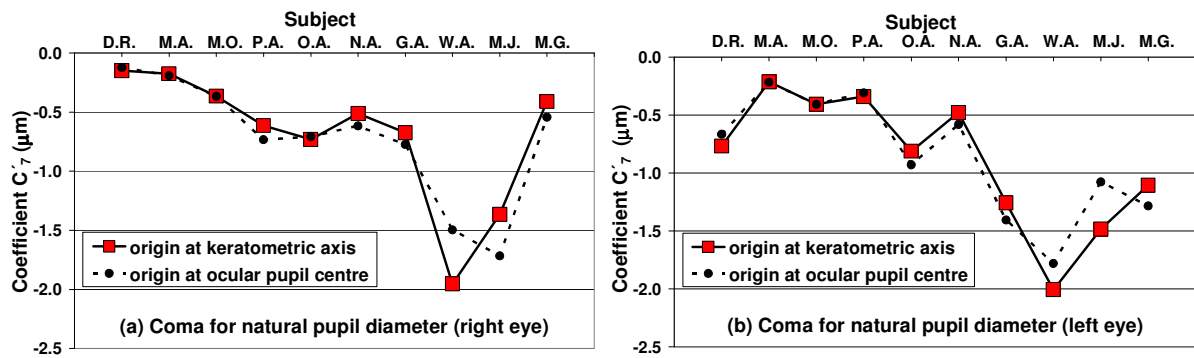


Figure 5 Variation of C'_7 when pupil centre is shifted from keratometric axis to ocular pupil centre

Besides, $C'_{7|F}$ can be greater or smaller than $C'_{7|0}$ (figure 5) and the difference $|\Delta C'_7| = |C'_{7|F} - C'_{7|0}|$ is small (this is, $|\Delta C'_7| < 0.15$ μm) for eyes in Groups I and II while it is up to 0.46 μm for those in Group III, this value corresponding to R of W.A. ($KISA\%^{(W.A.)} = 942$) whose pupil, pupil shift and coma are large ($DP^{(W.A.)}/2 = 2.3$ mm, $\delta^{(W.A.)} = 0.6$ mm and $C'_{7|0}^{(W.A.)} = -1.95$ μm). For example, we get the following.

- For both eyes of M.O. (Group I): $|\Delta C'_7|^{(M.O.)} = 0$
- For both eyes of G.A. (Group II): $|\Delta C'_7|^{(G.A.)} < 0.15$ μm
- For both eyes of M.G. (Group III): $|\Delta C'_7|^{(M.G.)} < 0.18$ μm

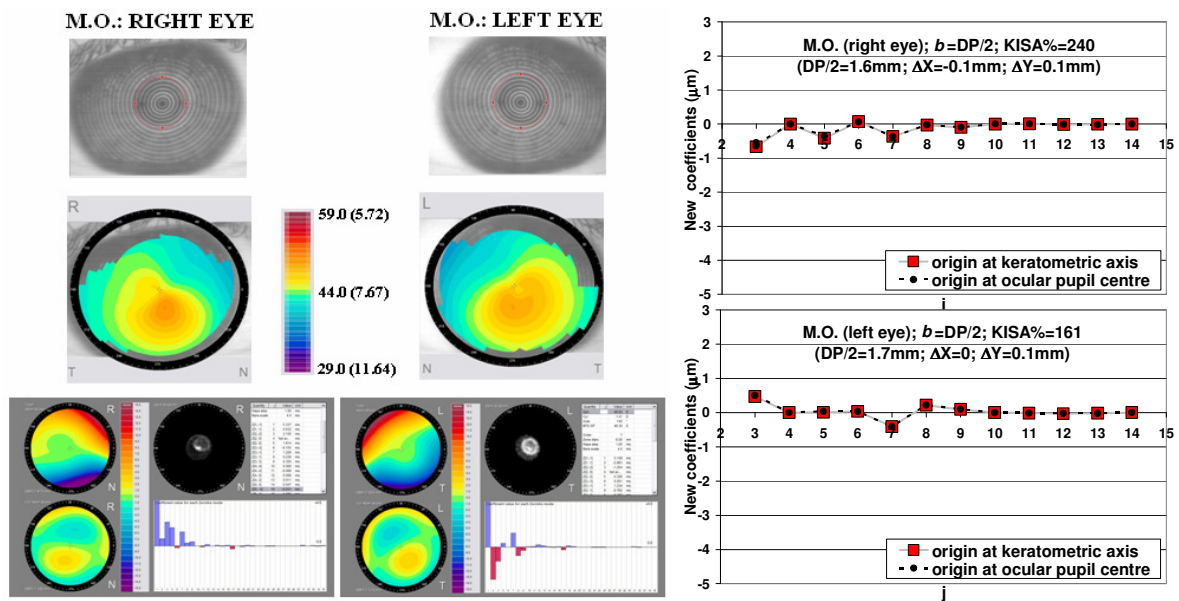


Figure 6 M.O. (Group I). Left: pupil, topography and corneal aberrometry (OPD, spot diagram and coefficients bar plot) if $a=3$ mm. Right: coefficients $C'_{j|0}$ and $C'_{j|F}$ ($3 \leq j \leq 14$) if $b=DP/2$ for R and L.

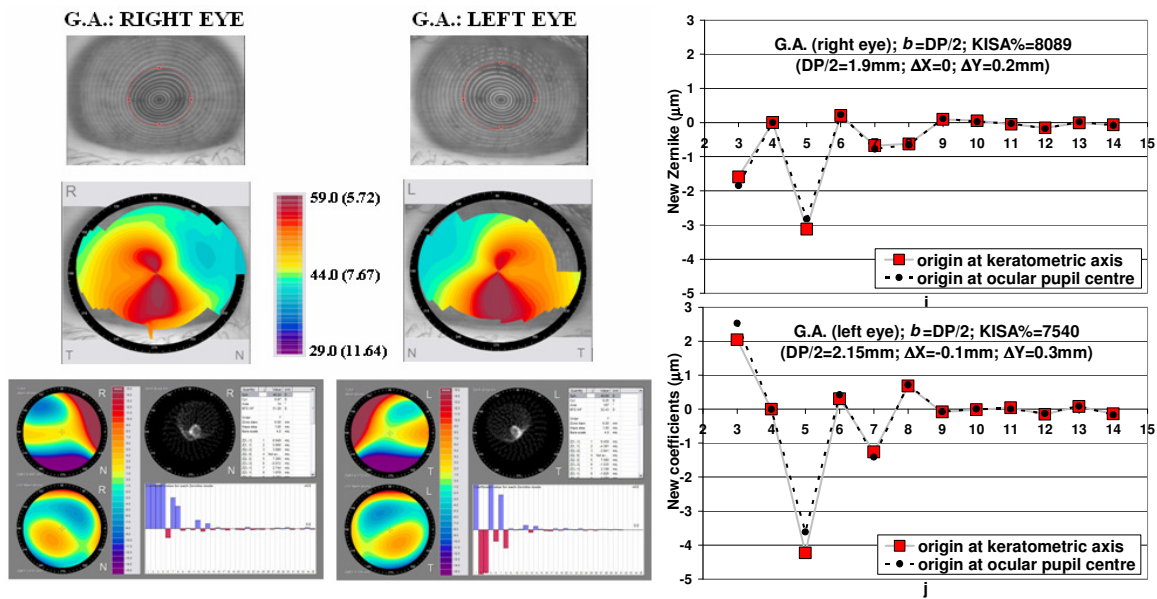


Figure 7 G.A. (Group II). Left: pupil, topography and corneal aberrometry (OPD, spot diagram and coefficients bar plot) if $a=3\text{mm}$. Right: coefficients $C'_{j|_0}$ and $C'_{j|_F}$ ($3 \leq j' \leq 14$) if $b=DP/2$ for R and L.

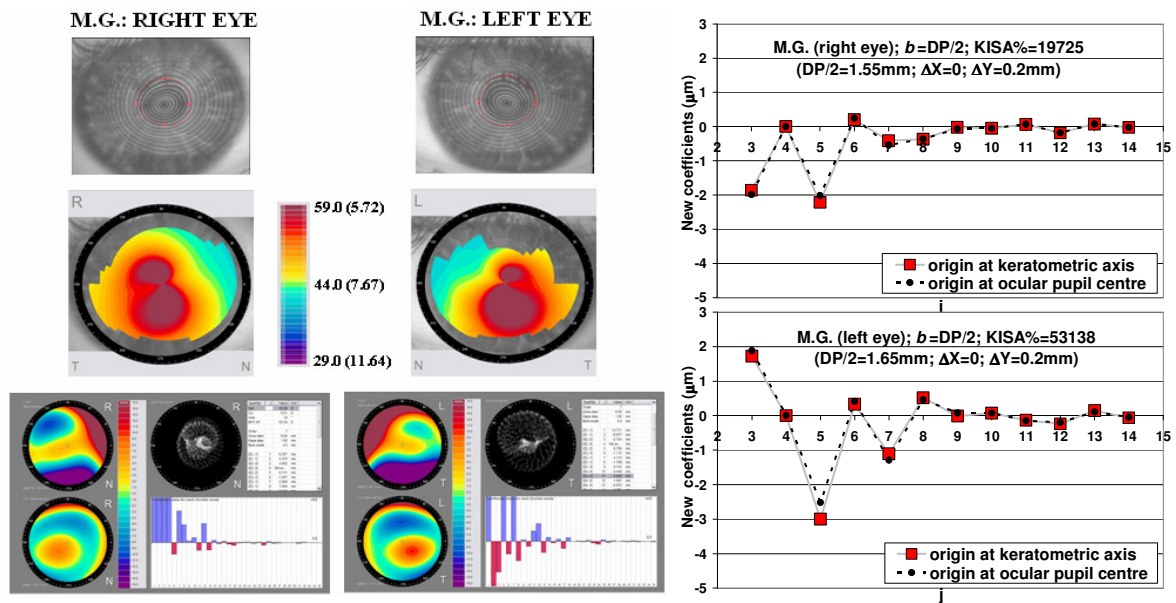


Figure 8 M.G. (Group III) Left: pupil, topography and corneal aberrometry (OPD, spot diagram and coefficients bar plot) if $a=3\text{mm}$. Right: coefficients $C'_{j|_0}$ and $C'_{j|_F}$ ($3 \leq j' \leq 14$) if $b=DP/2$ for R and L.

For these 3 subjects, figures 6, 7 and 8 show the pupil, the topography (revealing inferior temporal corneal steepening [18]) and corneal aberrometry for $a=3\text{mm}$ supplied by SN CT1000 (bar plots correspond to $OPD=-W$). Additionally, using the same scale, leaving aside as usual piston and tilt [7] and setting $C'_4=0$, we plot $C'_{j|_0}$ and $C'_{j|_F}$ (with $3 \leq j' \leq 14$) for the right (upper panel) and left (lower panel) eyes of each subject and obtain that $\Delta C'_{j'}$ is noticeable only for M.G. and for some values of j' .

Considering the 20 eyes, each with its natural photopic pupil size, we get that if aberrations are severe then $|\Delta\text{RMS}_{\text{HO}}| = |\text{RMS}_{\text{HO}|_{\text{F}}} - \text{RMS}_{\text{HO}|_{\text{0}}}|$ is up to $0.47 \mu\text{m}$ and, as reported by other authors [3,7], coefficients $C'_{j|_{\text{F}}}$ are more precisely evaluated using some methodology to transform coefficients. However if, as occurs for the first 6 eyes of figures 5 (a) and (b) (60% of the eyes considered here), we have $\text{RMS}_{\text{HO}|_{b=\text{DP}/2}} < 1.00 \mu\text{m}$ (moderate aberrations) then $|\Delta\text{RMS}_{\text{HO}}| < 0.12 \mu\text{m}$ and coefficients $C'_{j|_{\text{F}}}$ differ so little from $C'_{j|_{\text{0}}}$ that could be assumed to be those directly determined using the topographer. Notwithstanding this, it is interesting to point out that this assumption might not be applicable in so many eyes under non-photopic conditions since, in this case, pupils are often larger.

Conclusions

Considering 20 keratoconic eyes of 10 subjects (index KISA% ranging from 27 to 53138), we apply our Zernike coefficients transformation methodology to analyze how aberrations of the anterior corneal surface are modified when pupil parameters vary. Using the SN CT1000 topographer, we evaluate original coefficients of up to 7th order corresponding to a pupil of semi-diameter 3 mm centered at the keratometric axis, RMS_{HO} being up to $5.77 \mu\text{m}$. We obtain that RMS_{HO} is strongly related to index KISA% only for large pupils and that, as is well known, the dominant aberrations are 2nd order astigmatism and 3rd order coma, the largest higher-order coefficient corresponding to negative vertical coma. Under photopic conditions ((260 ± 20) lx in the eye analyzed by SN CT1000), the mean natural pupil semi-diameter is 1.8 mm and the ocular pupil centre is above the keratometric axis, the mean shift being 0.3 mm. Using our transformation formulas, we find new coefficients corresponding to each eye's photopic natural pupil. Coefficients computed considering coordinates origin at the ocular pupil centre can be greater or smaller than those evaluated assuming the origin on the keratometric axis. If the photopic pupil RMS_{HO} is greater than $1.00 \mu\text{m}$ then the difference between these coefficients is relatively large so, as stated by other authors, it is convenient to shift the coordinates origin to the ocular pupil centre and use a transformation methodology. However, if $\text{RMS}_{\text{HO}} < 1.00 \mu\text{m}$ (as occurs in 60% of the eyes considered here) then the difference between these coefficients is lesser than $0.12 \mu\text{m}$ (which is negligible according to the tolerance criterion for ocular aberrations) so corneal coefficients could be estimated with a tolerable error only using a topographer and without shifting coordinates origin. On the other hand, under non-photopic conditions, RMS_{HO} often increases (because pupil size increases), this simplification might not be so applicable and transformation formulas are probably required for more eyes.

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